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MOMENTUM ESTIMATION OF ELECTRONS TRAVERSING HEAVY MEDIA IN HOMOGENEOUS MAGNETIC FIELD II.

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MOMENTUM ESTIMATION OF ELECTRONS TRAVERSING HEAVY MEDIA IN HOMOGENEOUS MAGNETIC FIELD II.

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ABSTRACT

The method proposed in ref. [1] has been tested experimentally both on monochromatic positron beam traversing a heavy liquid bubble chamber and on γ rays produced in high energy interaction of K^+ mesons with the nuclei of a heavy liquid bubble chamber.

РЕЗЮМЕ

Метод, предложенный в работе [I], был проверен экспериментально с помощью пузырьковой камеры, наполненной тяжелой жидкостью, облученной монохроматическим позитронным пучком, а также с помощью гамма-квантов, рожденных в K^+ взаимодействиях с ядрами.

KIVONAT

Az [1] munkában javasolt módszer kisérleti ellenőrzését végeztük el nehéz folyadékkal töltött buborékkamrába besugárzott pozitron nyalábon és K^+ mag-kölcsönhatásban keletkezett fotonokon.

1. INTRODUCTION

In ref. [1] a new method was proposed for the determination of the momentum of fast electrons being detected in heavy media, e.g. in heavy liquid bubble chamber. In this paper we present some experimental results obtained by the proposed method. In Sec.2 we briefly recall the basic formulae to be used and introduce an approximation which facilitates considerably the application. Sec. 3 is then devoted to the description of the experimental procedure and to the presentation of the results. Some conclusions on the applicability of our method are also drawn in Sec. 3.

2. THE DESCRIPTION OF THE METHOD

As usual the components of the electron's three-momentum to be determined are replaced by the following new parameters:

a/ "dip" (λ), i.e. the angle between the electron's initial direction and the plane perpendicular to the magnetic field B, we call it /xy/ plane.

b/ "azimuth" (φ), i.e. the angle between the projected initial direction of the electron onto the /xy/ plane and the x axis chosen arbitrarily in this plane.

c/ , the initial curvature of the track projected onto the /xy/ plane, which, when measured in cm^{-1}, is connected to p , the absolute value of the electron's three momentum through the well known formula:

$$\rho_o = \frac{0.3B}{p.\cos\lambda} \quad . \qquad /1/$$

Here the magnetic field strength should be put in kGauss when p is measured in MeV/c.

Parameter /a/ can be estimated separetely whilst the estimation of ρ_o and ϕ is made in the same procedure, which introduces a correlation between their estimated values.

2.1.1. ESTIMATION OF THE DIP

For the best estimate of λ our method yields the following value:

$$tg\lambda_o = \left(\sqrt{b^2+4a^2} - b\right)/(2a) \; ;$$

$$\delta\lambda_o = \left(\frac{a}{\cos^2\lambda_o \; tg\lambda_o} \cdot \frac{1}{b^2+4a^2}\right)^{1/2} \qquad /2/$$

with

$$a = \bar{s}_i \; g_{ij}^{(1)} \; z_i$$

$$b = \bar{s}_i \; g_{ij}^{(1)} \; \bar{s}_j - z_i \; g_{ij}^{(1)} \; z_j \equiv b_1 - b_2$$

where $\bar{s}_i$ is the projected arclength of the i^{th} measured point, z_i its measured z-coordinate, and $g_{ij}^{(1)}$ is the inverse covariance matrix being the sum of two terms, one resulting from the Coulomb scattering, the other from the measurement error:

$$\left(g^{(1)}\right)^{-1}_{ij} = \left(\sigma_{cb}\right)_{ij} + \left(\sigma^{(1)}_{meas}\right)_{ij} = \langle d_i \; d_j \rangle \quad .$$

d_i is distance of the i^{th} point from the electron trajectory /see Fig. 1/.

It is interesting to compare equ./2/ with the result of the classical methods [2,3] :

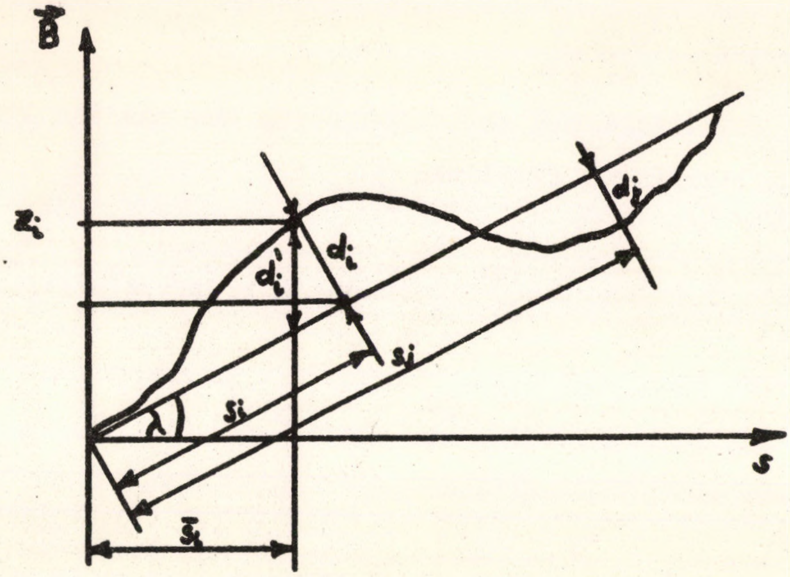

Fig. 1

$$tg\lambda_o = a/b_1$$
$$\delta\lambda_o = \cos^2\lambda_o/\sqrt{b_1} \quad . \qquad /3/$$

The two results are equivalent provided that in the latter case one replaces $\left(g^{(1)}\right)^{-1}_{ij} = \langle d_i' \, d_j' \rangle$, where d_i' is defined in Fig. 1. However, in practice one usually uses

$$\left(\sigma_{cb}\right)_{ij} = \frac{1}{12X_o}\left(\frac{21 \text{ MeV/c}}{p}\right)^2 s_i^2 \left(3s_j - s_i\right) \qquad /4/$$

where X_o is the radiation length of the liquid and s_i is defined in Fig. 1. Equ /4/ is correct in case of equ. /2/ but its use introduce an approximation /at least in principle/ for values obtained by equ. /3/. Clearly, this approximation is better when λ_o approches to zero, where equ. /2/ and /3/ coincide. Also, if $\sigma^{(1)}_{meas} >> \sigma_{cb}$, the two methods give practically the same results.

2.2. Estimation of ρ_o and ϕ

These parameters are estimated using the maximum likelihood method with the likelihood function:

$$P_{\perp}(\rho_o,\phi) = \int \exp\left\{- \frac{1}{2}\, n_i[\alpha(s)]\, g_{ij}^{(2)}\, n_j[\alpha(s)]\right\} . \mathcal{P}\, [\alpha(s)]\, \mathcal{D}[\alpha(s)] \;, \qquad /5/$$

where

$$n_i = y_i^m - y_i[\alpha(s)] \;,$$

y_i^m being the measured y-coordinate and

$$y_i \equiv y(s_i) = \int_0^{s_i} dT \sin\left\{\int_0^T \rho_o\, e^{\alpha(t)} dt + \phi\right\} \qquad /6/$$

is the so called Coulomb mean, determined by the continuously varying ionisation loss and by the randomly varying bremsstrahlung loss. $\rho_o \exp[\alpha(t)]$ describes the variation of the curvature and for a moment we took into account only the effect of bremsstrahlung due to which the initial energy, E_0 of the electron is absorbed by an absorbtion coefficient α and becomes:

$$E(t) = E_o\, e^{-\alpha(t)} \approx p_o\, e^{-\alpha(t)} = cst.\rho_o^{-1}\, e^{\alpha(t)} \quad .$$

is the electron's initiol momentum

Of course = 0 and t is a monotonously increasing function.

The integrand of equ. /5/ is thus factorised, the first factor giving the probability that y_i^m 's are observed once s is given, the second factor, on the other hand, stands for the probability of a particular trajectory. The likelihood function is then obtained by summing over all possible trajectories, this is meant by the symbol

As before,

Next we make the following approximation:

$$P_\perp(\rho_o,\phi) \approx \exp\left\{-\frac{1}{2}\, n_i[\bar{\alpha}(s)]\, g_{ij}\, n_j[\bar{\alpha}(s)]\right\}.\int \mathcal{P}[\alpha(s)].\, D[\alpha(s)] \quad . \qquad /7/$$

The last factor is known to be [4]

$$w(\alpha_o, s_o) = \frac{e^{-\alpha_o}\, \alpha_o^{bs_o - 1}}{\Gamma(bs_o)} \qquad /8/$$

with $b = \dfrac{1}{X_o \ln 2}$

and $\alpha_o \equiv \alpha(s_o)$,

s_o being the maximum tracklength.

$\bar{\alpha}(s)$ is a well defined trajectory, in our approximation a straight line:

$$\bar{\alpha}(s) = \alpha_o \frac{s}{s_o} \quad . \qquad /9/$$

This approximation is necessary, since the calculation of the original integral (5) cannot be carried out analitically and the numerical calculation usually requires considerable computer time. On the other hand the parametrisation of $\alpha(s)$ with only one parameter (c.f. equ. /9/) is justified in cases, where only few points are measured on the tracks, and thus the measured coordinates do not carry enough information on $\alpha(s)$. Moreover, as can be easily seen using the probability function /8/, $\bar{\alpha}(s)$ (function /9/) is the mean of all trajectories $\alpha(s)$.

Substituting equ./8/ and /9/ into equ. /7/, and taking into account in first order the deformation of the trajectory due to the ionisation loss, we get

$$P_\perp(\rho_o, \phi, \alpha_o) = \exp\left\{-\frac{1}{2}\left[\left(y_i^m - y_i\right) g_{ij}^{(2)} \left(y_j^m - y_j\right)\right] - \alpha_o\right\} . \alpha_o^{(bs_o-1)} \qquad /10/$$

where

$$y_i = \frac{1}{\Delta}\left\{\cos(\phi-\xi)\left[Si(t)\right]_{\xi}^{\xi e^{\Delta s_i}} + \sin(\phi-\xi)\left[Ci(t)\right]_{\xi}^{\xi e^{\Delta s_i}}\right\} + \frac{\kappa}{6\rho_o}\left(s_i\ \rho_o\right)^3$$

with $\xi = \frac{\rho_o\ s_o}{\alpha_o}$; $\Delta = \frac{\alpha_o}{s_o}$; $\kappa = \frac{2.2d}{0.3B}$,

d being the density of the liquid, and

$$Si(t) = \int \frac{\sin t}{t}\, dt \ ; \qquad Ci(t) = \int \frac{\cos t}{t}\, dt \ .$$

The best estimates of ρ_o and ϕ are the solutions of the system of equations:

$$\frac{\partial P_\perp}{\partial \rho_o} = \frac{\partial P_\perp}{\partial \phi} = \frac{\partial P_\perp}{\partial \alpha_o} = 0 \ . \qquad /11/$$

Inspecting equ. /10/ one can however recognise that the maximum likelihood method can only be applied, when

$$s_o > \frac{1}{b_o} \ , \qquad /12/$$

i.e. when the track length is long enough. Since in many practical cases condition /12/ is not fulfilled, we elaborated two methods for the case

$$s_o < \frac{1}{b_o} \ .$$

i/ The first method can be applied, if there are sufficient points on the track in order to carry information for the true value of α_o . The expression "sufficient" has a very qualitative meaning depending on the liquid, the distribution of the points on the track, etc. In practice the method is not applicable with less than six measured points.

If there are enough points, for some fixed values of α_o one resolves the system of equations

$$\frac{\partial P_\perp}{\partial \rho_o} = \frac{\partial P_\perp}{\partial \phi} = 0 \ . \qquad /13/$$

The obtained solutions have to be weighted by $\exp\left\{-\frac{1}{2}\times n_i(\alpha_o, \rho_o, \phi)\, g_{ij}\, n_j(\alpha_o, \rho_o, \phi)\right\}$, the probability of the "fit", multiplied by the probability á priori of the parameters [5], usually by $w/\alpha_o, s_o/$. The best estimate of the parameters, ρ_o, ϕ, α_o is their average weighted in this way.

ii/ On the other hand, if the measured points do not contain information on α_o /e.g. their number is not sufficient/ the first factor of the weight i.e. the probability of the fit does not depend on α_o, and the weighted average turns out to be always the same for all tracks. In many cases this can give rise to a systematic bias, e.g. to a systematic overestimate of the electron energy, as can be easily proved. Therefore in this case /practically always when the number of measured points is less than six /,we propose to choose α_o randomly according to distribution (8) and to solve eqs. /13/ with this value.

This procedure clearly increases the error of the parameters /in the case of gaussian distribution the error becomes $\sqrt{2}$ times larger/, but avoids systematic biases.

The errors on the parameters consist of two terms, the first being the usual one obtained at fixed α_o from the maximum likelihood method, the second is due to the fluctuation of α_o :

$$\delta p = \sqrt{\delta^2 p_{\alpha_o = \mathrm{fix}} + \left(\frac{\partial p}{\partial \alpha_o}\,\Delta\alpha_o\right)^2}\ , \qquad /14/$$

where

$$(\Delta\alpha_o)^2 = \int w(\alpha_1)\, w(\alpha_2)\, \delta(\alpha-\alpha_1-\alpha_2)\, \alpha^2\, d\alpha_1\, d\alpha_2\, d\alpha\ . \qquad /15/$$

This last integral is best to calculate by Monte Carlo method. The result is quoted in Table 1:

Table 1

α	0.	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
$(\Delta\alpha)^2$	0.	0.24	0.68	1.13	1.68	2.23	2.46	2.76	3.02	3.47

3. EXPERIMENTAL RESULT

3.1 Test on a monochromatic positron beam

In order to verify electron programs, D.Morellet has initiated the irradiation of the heavy liquid bubble chamber BP3 at Saclay by a monochromatic positron beam of momentum /464 $\pm$ 15/ MeV/c. The beam momentum was calibrated by measuring stopping protons inside the chamber, which had the same momentum as the positrons at the entrance. The radiation length of the liquid was 18 cm.

Each positron trajectory was then measured in 12 points distributed uniformly from the beginning of the track until that it rotated more than 60°. In 80% of the tracks the total measured length fulfilled the condition $b\ s_o > 1$. If not, we were practically always left with more than 6 points, so we used method i/.

In Fig.2 the calculated momentum distribution can be seen.

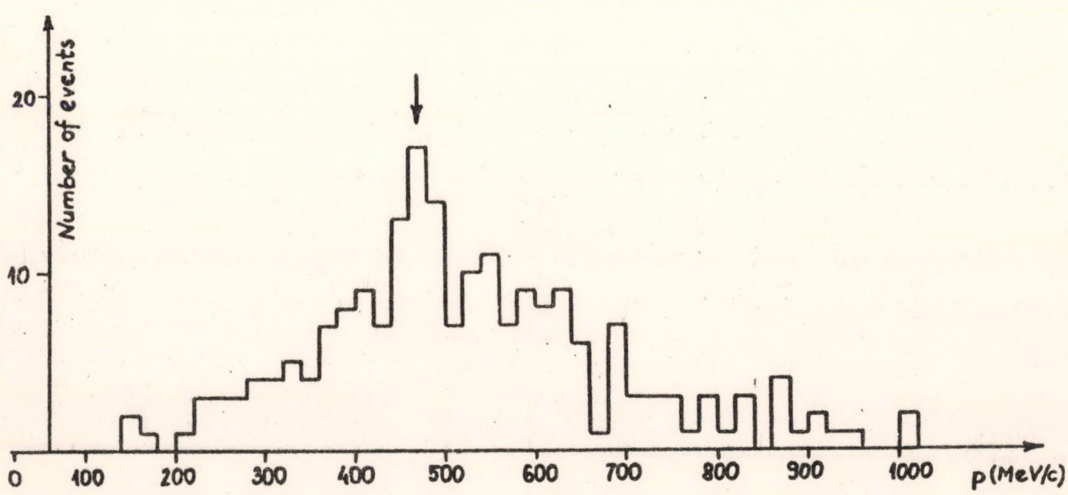

Fig. 2

The slash indicates the true energy. Events towards lower energies are mainly due to positrons which have lost energy at the entrance /before the first measured point/. On the other hand, the long tail towards higher energies is partly a consequence of our procedure, which in fact gives estimation on 1/p instead of p. This can be verified on the 1/p distribution /Fig. 3/, which is in fact nearer to a gaussian one.

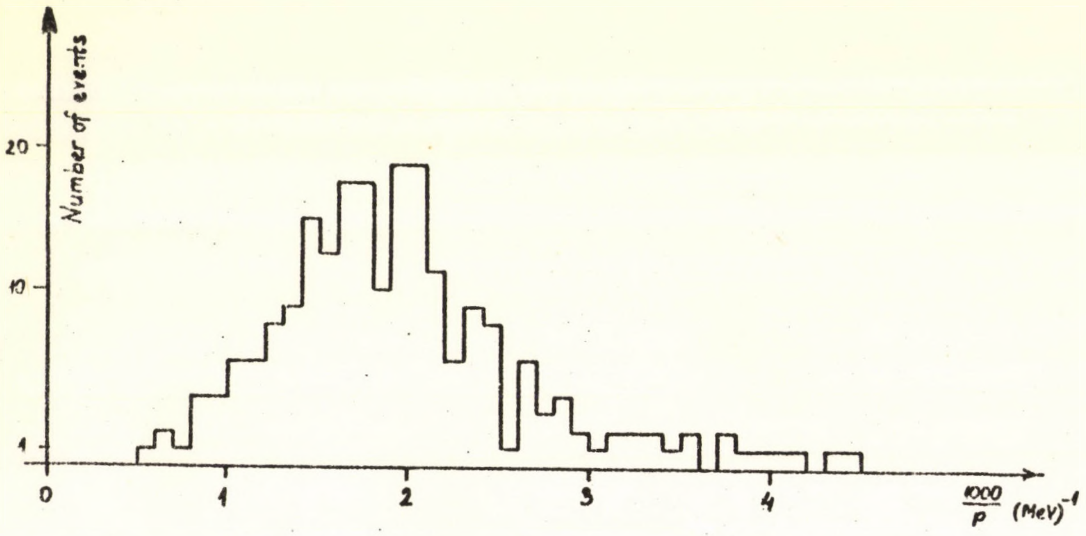

Fig. 3

Therefore, in order to estimate the average relative error of our momentum determination we used the half width of the 1/p distribution:

$$\left\langle \frac{\Delta p}{p} \right\rangle \approx \frac{\left\langle \Delta \frac{1}{p} \right\rangle}{\left\langle \frac{1}{p} \right\rangle} = 27 \% \ .$$

In Fig. 4 we plot the distribution of the relative error calculated for each individual positron. This estimation seems to be correct in comparing it with the average value given above.

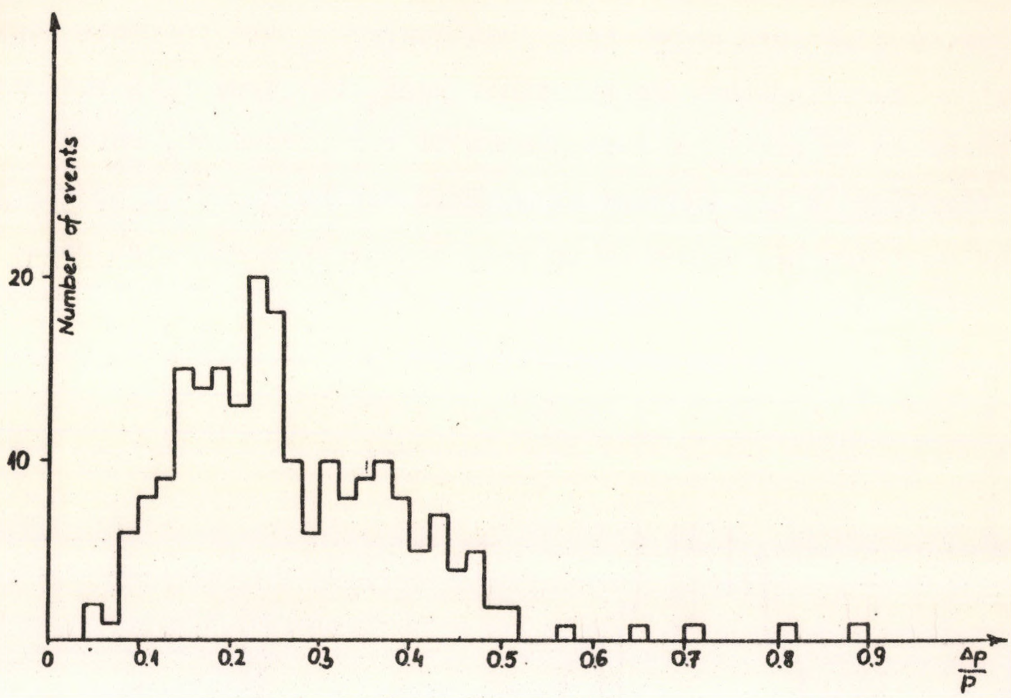

Fig. 4

3.2 Test on π^o production

As a second test of our electron program we used γ -rays from the $K^+ \ N \rightarrow K^+ \ N' \ \pi^+ \ \pi^- \ \pi^o$ reaction studied in the CERN 1 m heavy liquid bubble chamber [6]. The liquid composition was 60% $C_3 \ H_8$ and 40% CF_3Br with radiation length 25 cm and density 0,83 g/cm^3. The events were taken if two and only two γ rays were seen associated with a 3-prong interaction.

Since these two photons were produced with a high probability in a π^o decay, our first aim was at the reconstruction of the two photon invariant mass distribution.

The number of the measured points on the electron and positron trajectories varied from 3 to lo /Table 2/. We have taken the points on the track until that it has not been curved more than 60^o.

Table 2

Number of points on tracks	3	4	5	6	7	8	9	1o	Σ
Number of tracks	231	351	53	163	62	25	76	77	1o44

For 3 point tracks the momentum was determined by calculating the radius of a circle connecting the 3 points and then simply correcting it for bremsstrahlung and ionisation loss. Almost in every case, where bs_o turned out to be smaller than 1 /in 63% of the tracks/, we applied our method ii/ i.e. we have chosen α_o randomly. The effective mass distribution obtained in this way is shown in Fig.5. The π^o

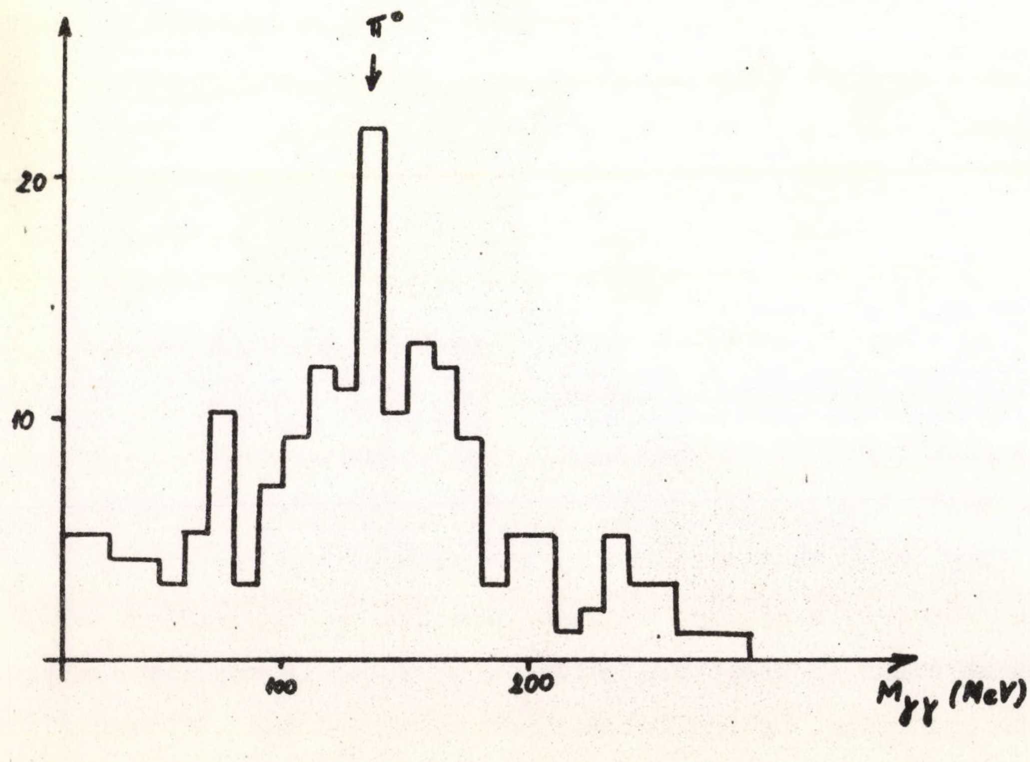

Fig. 5

peak is located on the correct place. The background to the peak consists mainly of events, where the γ's were not produced in a single decay.

For the sake of comparison in Fig. 6 the same events are shown,

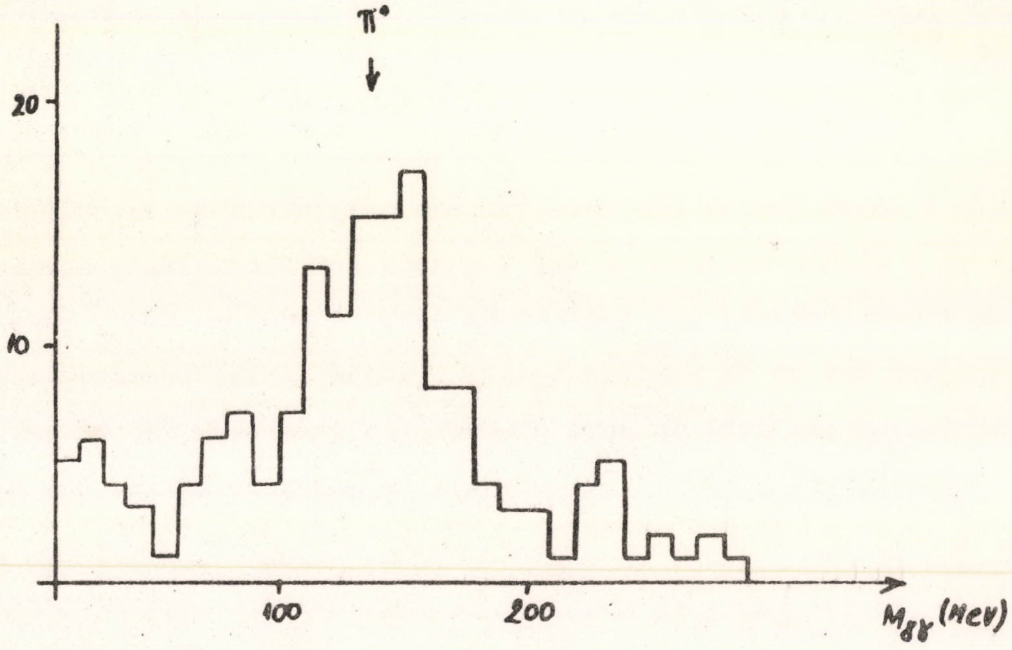

Fig. 6

where in the case of $b\ s_0 < 1$ we applied method i/. The average error seems to be slightly less but the π^0 peak is slightly shifted towards higher energies, just as expected.

The same material enabled us to verify our estimation concerning ϕ as well. Namely, we could compare the calculated direction of every γ ray with that obtained by the measurement of the interaction point /the origin of the γ rays/ and the γ ray vertex. Since the measurement error of the coordinates in question is usually small, the direction determined by the interaction point and the γ vertex can be considered as "true". In Fig. 7 we plot the distribution of the

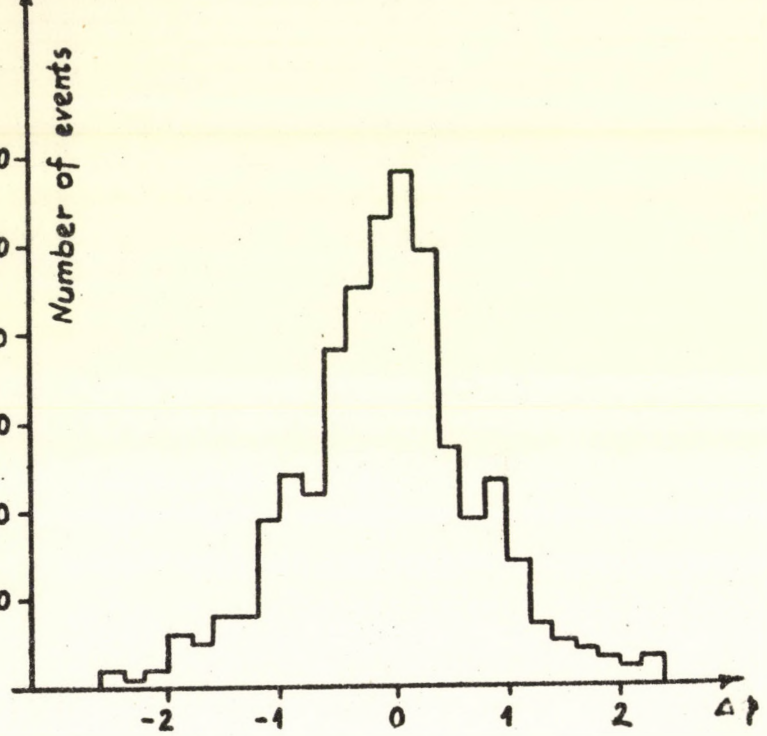

Fig. 7

quantity

$$\Delta_\phi = \frac{\phi^{\text{"true"}} - \phi^{\text{measured}}}{\sigma_\phi}$$

where σ_ϕ is the estimated measurement error. If the estimation is correct, the quantity Δ_ϕ should have a gaussian distribution of unit halfwidth. The observed distribution fulfils this criterion fairly well. We have obtained a similar distribution for the dip, λ .

As a last step we compare our dip estimation /equ /2/ /with the classical one /equ /3/ / in Fig. 8. No significant difference can be observed, which is explained mainly by the fact, that the measurement errors of the z -coordinates are rather big.

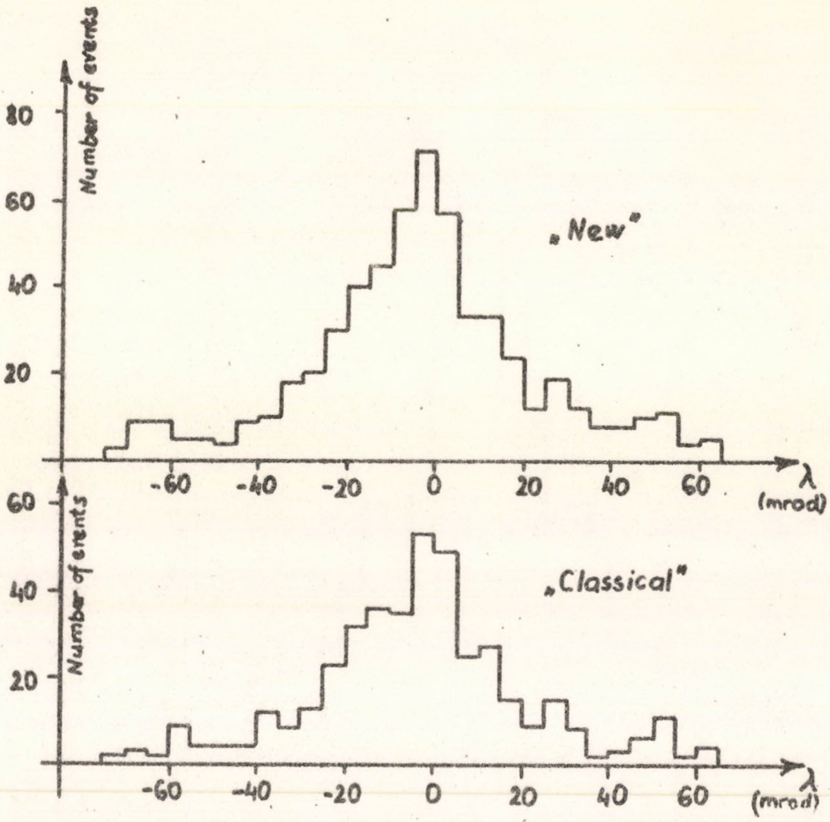

Fig. 8

In conclusion let us summarize the results of our experimental verification. Our direction estimation seems to be correct. The average relative error of the energy estimation is 27% in a liquid of 18 cm radiation length. /The corresponding value obtained by the SPIRAL method [1] is very near to this one/. Beside these facts we should like to point out the special virtue of our method mainly of practical use: it rejects a very few per cent /in general 1-2%/ of tracks as unestimable.

In our opinion the main part of uncertainty in our energy estimation is due to the approximation made in deriving equ /7/. Therefore we have tryed to approximate better the actual trajectory $\bar{\alpha}$ s than with a linear one by introducing more parameters. However, immediately we were faced with two difficulties: 1. The new pocedure requires more measured points on the track, and 2. an order of magnitude more computer time. So it cannot be used when a great number of events has to be treated.

I should like to express my deep gratitude to Prof. A. Lagarrigue for his warm hospitality at the Orsay Laboratoire de l' Accélérateur Linéaire, where a part of this work was carried out. I am indebted to Dr. D. Morellet, who kindly lent me the experimental material on positron tracks and thus enabled me to carry out a verification of the proposed method. I am also indebted to him for useful discussions. A second verification of the method was carried out on an experimental material kindly send me by R. Arnold, which is greatly aknowledged.

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Annexe I.

[6] Paris - Bergen - Strasbourg - Madrid Collaboration.
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